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A method for determination and control of the frequency response of the constant-temperature hot-wire anemometer

By N. B. WOOD
Central Electricity Research Laboratories, Leatherhead, Surrey, England

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The theory of the constant-temperature anemometer has been extended in order to obtain quantitative results for the frequency response. A simple electrical test against which to check the theory has been devised, and the validity of the anemometer equations is demonstrated. Important differences in design philosophies and modes of operation are indicated, and results are presented for a design in which high d.c. gain is employed in the servo amplifier. The square-wave response is briefly investigated, and it is concluded that commonly used criteria for determining the frequency response from it should be treated with caution.

To measure fluctuations with a hot-wire anemometer in flows containing both velocity and temperature perturbations, the hot wire must be operated at more than one temperature. Variation of the mean wire temperature causes, in general, a variation in the frequency response, as does variation of the mean flow conditions. It is shown that, by simultaneous variation of the gain of the servo amplifier in the anemometer, the frequency response may be held nearly constant over a useful range of both overheat and flow conditions.

1. Introduction

The work described here was undertaken to facilitate the measurement of fluctuations in high speed flows through turbines, specifically those occurring in the low pressure cylinders of steam turbines, although the results obtained are general. The scheme adopted for the turbine measurements follows the method of Kovasznay (1950) and the present application has been fully discussed by Wood (1973). In order to separate the fluctuations in mass flow and temperature, the relative sensitivity of the anemometer to the two components is varied by varying the overheat. An alternative technique used more recently (e.g. Tennekes & McCall 1971) involves continuous simultaneous measurements with two wires operated at widely differing overheats and on-line automatic data analysis.

In either case, the frequency response of a constant-temperature anemometer will, in general, be different for different overheats and different flow conditions, and if the frequencies present are high, as in high speed flows, large errors may result. If a compensated constant-current anemometer is used, as in the case of Kovasznay, the problem does not arise because the response is compensated at each condition. In flows where fluctuations may be large, however, as in turbines, the constant-current anemometer is not appropriate. Therefore, use of the constant-temperature system may be desirable, and for a high speed environment the frequency response must then be determined for each overheat and flow condition so that, if necessary, it may be compensated or controlled.

2. Problems of determining frequency response

If a theoretical analysis of the constant-temperature anemometer can be shown to give sufficiently reliable results, then the frequency response may be obtained rapidly from this theory, via a computer. The tedium of a large number of measurements of the frequency response for all conditions of interest is thus avoided. In any case, there is considerable difficulty in making accurate measurements of the frequency response by any of the techniques described in the literature.

In setting up a constant-temperature anemometer, it is a standard procedure to apply a square-wave test signal across the probe, or across some other part of the bridge. The resulting system response is monitored, and adjustments are made to obtain an optimum signal, representing fast frequency response but without instability. Rules are sometimes given for obtaining the frequency response from the square-wave test response but, as shown in this paper, these methods are either inappropriate or unreliable.

In an alternative indirect technique, Nielsen & Rasmussen (1966) used a sinusoidal electrical perturbation in place of a square wave. The output was measured at different frequencies and compared with the results obtained when the hot wire was replaced by a dummy probe with a long time constant. However, the equations used to transform these results into the velocity fluctuation response are ill conditioned in the region of the roll-off frequencies, and wildly scattered results can be obtained.

Measuring the response directly by placing the wire in a vortex-street cylinder wake (Grant 1968) seems attractive, but extreme care is required in setting this up if accurate results are to be obtained, particularly at higher frequencies. The complication of wire vibration (Perry & Morrison 1971*b*) may also arise.

A technique which may be categorized as lying between the direct and indirect methods is the use of audio-frequency modulated microwaves directed at the wire (Kidron 1966). However, the equipment necessary may not be widely available in laboratories where hot-wire measurements are made. Further, the microwave energy absorbed by the wire may alter the operating resistance. Although the overheat may be maintained by measuring the cold resistance in the microwave beam, the change in both the cold and operating resistances may have a significant effect on the frequency response.

In the present case, where it was desired to test the theory, measurements were required which were simple to make and which used laboratory equipment which was generally available. A method which meets these conditions is the measurement of the response to sinusoidal electrical perturbations applied across the wire. The theory may be used to predict this response and, if successful, it may then be used for the prediction of the response to velocity perturbations or, more generally, to heat-transfer perturbations.

It appears that the theory of constant-temperature anemometers has not previously been subjected to such a detailed test.

3. Theory of the constant-temperature anemometer with an inductive bridge

Early papers on the theory of constant-temperature hot-wire anemometers (Weske 1943; Ossofsky 1948; Janssen, Ensing & van Erp 1959; Grant & Kronauer 1962) considered only the effect of the wire time constant and amplifier characteristics, that is, they neglected the effects of bridge reactance. Freymuth (1967) introduced the effect of capacitive bridge reactance; Davis & Davies (1968) and Davis (1970) included probe cable inductance, pointing out that in most circuits its effect dominates. In fact, it is difficult to envisage a practical hot-wire circuit having a non-inductive bridge. The analyses of Davis & Davies and Davis were for circuits with inductive compensation on the active side of the bridge. The more common arrangement, in which the inductive compensation is incorporated into the passive side of the bridge, was considered by Perry & Morrison (1971*a*) and is given further attention here.

The present paper discusses a constant-temperature anemometer system which employs a different design philosophy from that of Perry & Morrison, and whose behaviour differs in some respects from that described by them. Perry & Morrison discussed a design in which the bridge was allowed to be out of balance at the wire operating point, and the system was largely controlled by an amplifier input bias (or offset) voltage adjustment. In the present case an extension of this theory is applied to a system which is forced into balance by a very high d.c. gain in the amplifier. The system is then controlled by a combination of a.c. gain variation and inductive bridge compensation. The high d.c. gain, as will be shown, nullifies the effect of amplifier bias on the system performance, and the anemometer is, in fact, much simpler to operate. (Perry & Morrison reported comparisons between a crude system of this type and their own system, but they appeared to misinterpret its mode of operation.)

Figure 1 shows the usual arrangement of the anemometer circuit; both in the diagram and in the analysis which follows, the main notation is that used by Perry & Morrison in their linearized small perturbation analysis. The hot-wire probe and cable, with inductance L_W, form one arm of the bridge. The operating resistance of the hot-wire element is R_W. In addition, the resistance R_L of the probe leads and cable must be distinguished from the active resistance, and taken into account. These resistances are balanced on the passive side of the bridge by the resistance R_t of the trim potentiometer and the resistance R_b of the overheat-setting resistors. The probe inductance is compensated on the passive side of the bridge by the inductance L_b of the variable inductor.† Stability may be assisted in some systems by a d.c. bias E_{qi} at the amplifier input, and sometimes an output

† The bridge also contains capacitances, but the effect of these is negligible below approximately 1 MHz.

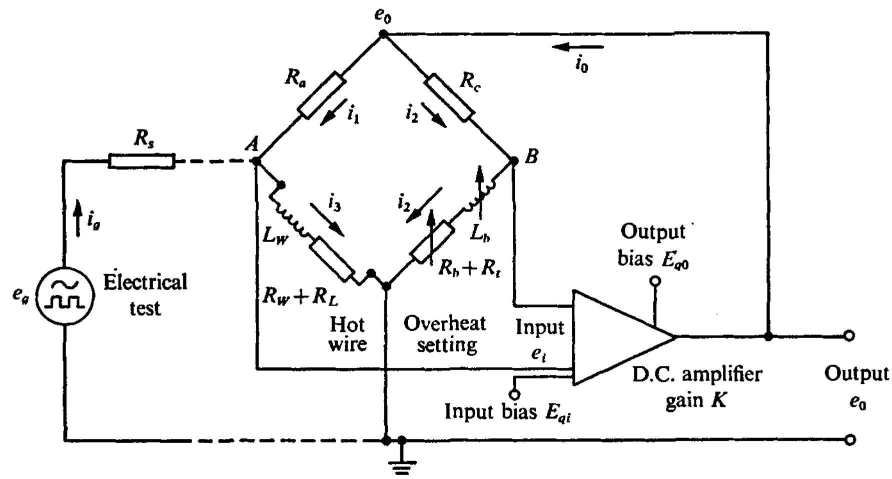

FIGURE 1. Block diagram of constant-temperature hot-wire anemometer.

bias is also provided. The bias, or offset, in combination with the amplifier gain may also affect the wire operating resistance and the system frequency response, as discussed by Perry & Morrison (1971*a*).

In the equations given by Perry & Morrison, the effect of the trim and lead resistances may be included by writing

$$R_{WL} = R_W + R_L, \quad R_{bt} = R_b + R_t \tag{1), (2}$$

and replacing R_b by R_{bt} and R_W by R_{WL} in their equations, except in the ohmic heating term or where R_W appears in the combination $R_W - R_g$, R_g being the equilibrium or 'cold' resistance of the wire. A typical value for R_L (leads plus cable) would be 1 Ω, and since the commonly used 5 μm diameter tungsten wires have a room-temperature resistance of 3–4 Ω per mm, the lead resistance is obviously significant. In the tests described here, in which a high resistance cable was used (Wood 1973), R_L was greater than 2 Ω.

A further factor which was omitted from the analysis of Perry & Morrison was the effect of finite bandwidth of the amplifier, although it was briefly discussed. Whilst a modern amplifier is likely to have a high enough frequency response for its effect to be ignored for most purposes this effect cannot necessarily be dismissed as lightly as was done by Perry & Morrison, and the bandwidth is therefore included here.

In the system used in the present experiments (DISA 55D01) the amplifier had a second-order characteristic, i.e. its behaviour could be described by the equation

$$Ke_i = (M''D^2 + M'D + 1)\, e_0, \tag{3}$$

where e_i and e_0 are the small perturbation input and output voltages, K is the a.c. gain and M' and M'' are the first- and second-order time constants. D is the differential operator d/dt. This form of amplifier behaviour was also assumed by Freymuth (1967) and by Grant & Kronauer (1962).

Equations (26)–(28) of Perry & Morrison (1971 *a*) reduce to the simple equation

$$e_0 = Ke_i \tag{3a}$$

although the accompanying description tends to obscure this fact. Therefore, these equations are replaced here by the present equation (3). Combining this with equations (1)–(25) of the original analysis, but allowing for the inductive elements in the probe arm by writing

$$e_0 = -\frac{u'x}{TD+1} + i_1\left[R_a + R_{WL} + L_W D + \frac{\alpha}{TD+1}\right] \tag{4}$$

(where u' and x are defined as in Perry & Morrison 1971*a*) and for those in the compensation arm of the bridge by writing

$$e_0 = (L_b D + R_{bt} + R_c)\, i_2, \tag{5}$$

the following equation for the system results:

$$\frac{e_0}{u'} = \frac{K_1(E_1 D + 1)}{A_5 D^5 + A_4 D^4 + A_3 D^3 + A_2 D^2 + A_1 D + 1}, \tag{6}$$

where $K_1 = KR_a(R_{bt} + R_c)\, x/R_K,$ (6*a*)

$$R_K = (R_{bt} + R_c)(R_a + R_{WL} + \alpha) + K(\mathring{R} + R_c\alpha), \tag{6b}$$

$$E_1 = L_b/(R_{bt} + R_c), \tag{6c}$$

$$A_5 = M'' L_b L_W T/R_K, \tag{6d}$$

$$A_4 = \{(M'' + M'T) L_b L_W + M''T[L_W(R_{bt} + R_c) + L_b(R_a + R_{WL})]\}/R_K, \tag{6e}$$

$$\begin{aligned} A_3 = \{&(M'' + M'T)[L_W(R_{bt} + R_c) + L_b(R_a + R_{WL})] \\ &+ M''T(R_{bt} + R_c)(R_a + R_{WL}) + M'' L_b \alpha + (M' + T) L_b L_W\}/R_K, \end{aligned} \tag{6f}$$

$$\begin{aligned} A_2 = \{&(M'' + M'T)(R_{bt} + R_c)(R_a + R_{WL}) + M''\alpha(R_{bt} + R_c) \\ &+ (M' + T)[L_b(R_a + R_{WL}) + L_W(R_{bt} + R_c)] \\ &+ L_b(M'\alpha + L_W) + KT(R_c L_W - R_a L_b)\}/R_K, \end{aligned} \tag{6g}$$

$$\begin{aligned} A_1 = \{&(R_a + R_{WL} + \alpha)[M'(R_{bt} + R_c) + L_b] \\ &+ (R_{bt} + R_c)[T(R_a + R_{WL}) + L_W] \\ &+ K(R_c L_W - R_a L_b + \mathring{R}T)\}/R_K, \end{aligned} \tag{6h}$$

$$\alpha = 2R_W(R_W - R_g)/R_g, \tag{6i}$$

$$T = C(R_W - R_g)/I_1^2 R_g \quad \text{(see (8) below)}, \tag{6j}$$

$$\mathring{R} = R_{WL} R_c - R_a R_{bt}. \tag{6k}$$

It is convenient to write the equation in terms of a perturbation h of the wire heat-transfer coefficient H in combination with the steady bridge voltage E_0. This forcing function was adopted by Freymuth (1967) in his analysis of a system with a capacitive bridge, and is more useful when compressible flow measurements are being considered.

If (6) is written as

$$\frac{e_0}{u'x} = \frac{K_2(E_1D+1)}{f_L(D)}, \tag{7}$$

where $K_2 = K_1/x$ and $f_L(D)$ is the denominator of the right-hand side, $u'x$ may be replaced as follows. The wire heat-transfer equation [equation (15) of Perry & Morrison] may be written as

$$I_1^2 R_W = H(T_W - T_g) + m\frac{dT_W}{dt} = \frac{H}{\beta}(R_W - R_g) + C\frac{dR_W}{dt}, \tag{8}$$

where $C = m/\beta$ and $\beta = (R_W - R_0)/(T_W - T_0)$, R_0 and T_0 being the resistance and temperature at reference conditions.† (m is the thermal capacity of the wire.) In small perturbation form this becomes

$$e_W = -\frac{(R_W - R_g)^2}{I_1 R_g}\frac{h}{\beta}\Big/(TD+1), \tag{9}$$

so that for $u'x$ in (7) is substituted $(R_W - R_g)^2 h/I_1 R_g \beta$ and the system equation becomes

$$\frac{e_0/E_0}{h/H} = \frac{R_W(R_W - R_g)}{R_g(R_{WL}+R_a)}\frac{K_2(E_1D+1)}{f_L(D)} = \frac{K_3(E_1D+1)}{f_L(D)}. \tag{10}$$

3.1. *Response of the anemometer to applied electrical perturbations*

Important characteristics of the performance of the anemometer may be deduced by examination of the response to electrical perturbations, which may be of square-wave or sine-wave form. The perturbation may be applied to the amplifier bias (Perry & Morrison 1971*a*), at the amplifier input (Davis & Davies 1968), across the compensation arm of the bridge (Thermosystems Inc.) or across the probe (DISA). The experiments reported here were carried out with a DISA anemometer; therefore the following analysis is applied to the latter system, although the principles involved are similar in all cases. Perturbation of the bias would, however, have no effect on a system like the DISA, which uses very high d.c. gain.

Referring to figure 1, the equations for the currents in the various elements of the circuits are as follows:

$$i_0 = i_1 + i_2, \quad i_3 = i_1 + i_g, \quad i_2 = e_0/Z_2, \qquad (11)\text{–}(13)$$

$$e_0 - e_A = i_1 R_a, \quad e_0 - e_B = i_2 R_c, \quad e_g - e_A = i_g R_s, \qquad (14)\text{–}(16)$$

$$e_A = i_3 Z_W, \quad e_B = i_2 Z_b, \quad e_i = e_B - e_A, \qquad (17)\text{–}(19)$$

where $$Z_W = R_{WL} + L_W D + \alpha/(TD+1), \quad Z_b = R_{bt} + L_b D.$$

† It may be noted that the equation for the wire is written in terms of 'lumped' characteristics. Davis & Davies (1968) showed that the effect of unsteady conduction in the wire may be considered separately; it does not affect the feed-back system, which maintains the mean wire resistance constant. The spatial resolution of the wire may also be considered separately (Roberts 1973).

(N.B. This is not the same as the bridge impedance Z_B introduced by Perry & Morrison.)

$$Z_1 = R_a + Z_W, \quad Z_2 = R_c + Z_b.$$

Combining (14), (17) and (12) gives

$$i_1 = (e_0 - i_g Z_W)/(R_a + Z_W). \tag{20}$$

From (16) and (14),

$$i_1 = (e_0 - e_g + i_g R_s)/R_a. \tag{21}$$

Eliminating i_1 from (20) and (21) yields

$$i_g = (e_g Z_1 - e_0 Z_W)/(R_s Z_1 + R_a Z_W). \tag{22}$$

Substituting (13) into (18), and the result with (16) into (19) gives

$$e_i = e_0 Z_b / Z_2 - e_g + i_g R_s. \tag{23}$$

Substituting (22) into (23) gives

$$\begin{aligned} e_i &= \frac{e_0 Z_b(R_s Z_1 + R_a Z_W) - Z_2 R_s Z_W}{Z_2(R_s Z_1 + R_a Z_W)} - e_g \frac{R_a Z_W}{R_s Z_1 + R_a Z_W} \\ &= (e_0/K)(M'' D^2 + M' D + 1) \quad \text{[from (3)]}. \end{aligned} \tag{24}$$

Expanding (24) and putting $R_s \gg R_a$ yields

$$\frac{e_0}{e_g} = -\frac{K_2(E_1 D + 1)}{R_s f_L(D)} [(TD+1)(R_{WL} + L_W D) + \alpha]. \tag{25}$$

Equations (10) and (25) may be solved for a harmonic forcing function by substituting $(i\omega)^n$ for D^n, which reduces them to algebraic equations. In the case where e_g is a square wave, the response to each step may be solved by obtaining the Laplace transform of (25), with $\bar{e}_g = E_g/p$, E_g being the magnitude of the applied voltage step, p being the Laplace variable and the overbar representing the transformed variable. The transformed equation is thus

$$\frac{\bar{e}_0}{E_g} = -\frac{(K_2/R_s)(E_1 p + 1)}{p f_L(p)} [(Tp+1)(R_{WL} + L_W p) + \alpha]. \tag{26}$$

The result in the real domain may be obtained from a table of transforms (e.g. Starkey 1958).

3.2. *Effect of high d.c. gain*

If the d.c. gain of the amplifier is made sufficiently high, the bridge is forced very nearly into balance ($R_{WL}/R_{bt} \doteq R_a/R_L$), so that the operating point is fixed at the value set by R_b, and $\mathring{R} = R_{WL} R_c - R_a R_{bt} \doteq 0$. For example, if the wire mentioned by Perry & Morrison (1971*a*; $R_g = 5\,\Omega$, nominal overheat $= 2{\cdot}0$) is connected to a DISA 55D01 anemometer with $K_{dc} = 45\,000$, the balance point ($\bar{R}_W = 10\,\Omega$) is achieved with an input bias $E_{qi} = 60{\cdot}9\,\mu$V. A cross-plot of equations (5) and (6) from the paper by Perry & Morrison gives operating points (values of $\bar{R}_W$) of 9·99919 for $E_{qi} = 30\,\mu$V and 10·00102 for $E_{qi} = 100\,\mu$V. Thus, moderate variation of the d.c. bias of the amplifier, as discussed by Perry &

Morrison, has a negligible effect on the operating point in this case and a small effect on the frequency response.

However, if the bias is reduced towards zero, $\mathring{R}$ becomes increasingly negative, although it approaches a limiting value. Examination of equation (5) of Perry & Morrison shows that the d.c. balance of the system may then be upset, causing it to cease operating. If the a.c. gain is sufficiently high, there will also be a significant decrease in the damping.

In the case of the DISA 55D01 anemometer, if the bias controls are centred as recommended the system will operate sufficiently close to balance for the effect of bias to be ignored. This mode of operation makes the anemometer system much simpler to operate, since the calibration does not alter with flow conditions, although it is necessary to alter the tuning if optimum frequency response is always required. Since it is this type of anemometer which is the subject of the paper, it will be assumed throughout the following discussion that $\mathring{R} = 0$.

It should be noted that the theory presented here for the a.c. response holds only for frequencies above that at which the a.c. gain becomes flat (6–210 Hz in the DISA 55D01, depending on the a.c. gain setting). The a.c. response at this frequency will be lower than the d.c. response, but the discrepancy will only be significant, if at all, at low overheat, combined with low gain. For example, in the worst of the cases cited here, an overheat of 1·1 with an a.c. gain of 1100, the a.c. response at 100 Hz is 0·93 of the d.c. response.

If it is desired to calculate the response in the frequency range where the gain is falling from its d.c. level to the a.c. plateau, then an additional term would have to be included in (3), involving the time constant of the amplifier feed-back loop which determines the low frequency gain.

4. Interpretation of anemometer equation

Before embarking on numerical computations of (10), (25) and (26), we may deduce some general features of the system behaviour. Some aspects of the frequency response and stability of the anemometer may be obtained from a consideration of the system's poles. For stability over the whole bandwidth, the poles should have negative real parts, and a number of techniques is available for estimating the degree of stability (see, for example, Horowitz 1963).

Examination of the magnitudes of the poles shows that, for practical cases, two of the five poles are large compared with the others, so that the important characteristics of the system response will be dominated by the remaining three poles. Therefore the anemometer equation may be reduced to third order, and calculation of the magnitudes of the terms contained in the coefficients A_1, A_2 and A_3 indicates that they may be simplified as follows:

$$\left.\begin{aligned} A_3 &\simeq M''T(R_{bt}+R_c)(R_a+R_{WL})/R_K, \\ A_2 &\simeq [M'T(R_{bt}+R_c)(R_a+R_{WL})+KT(R_cL_W-R_aL_b)]/R_K, \\ A_1 &\simeq T(R_a+R_{WL})(R_{bt}+R_c)/R_K, \\ R_K &\simeq KR_c\alpha. \end{aligned}\right\} \quad (27)$$

(These approximations have been introduced solely to facilitate the following discussion; the full equations have been programmed to produce the results shown in the figures.)

If the effects of bridge inductance and amplifier bandwidth are neglected, a first-order equation results, and is a simplified version of equation (29) of Perry & Morrison (1971*a*), i.e.

$$\frac{e_0/E_0}{h/H} = K_3\bigg/\left(\frac{T(R_a+R_{WL})\,(R_{bt}+R_c)}{KR_c\alpha}D+1\right). \tag{28}$$

Recalling that T is the constant-current wire time constant, the role of the gain in reducing this time constant in the constant-temperature anemometer is seen immediately. For the system under consideration, the coefficient of D is approximately $10T/K$, so that to improve the frequency response by one or two orders of magnitude gains of 100 and 1000 respectively are required.

4.1. *Optimum response*

Returning to the third-order equation (27), it is found that for a well-tuned system, with optimum frequency response and damping, one pole is large compared with the remaining quadratic pair, which may be complex conjugates or real. The large real pole is approximated as $S_3 = -A_2/A_3$ and the equation for the quadratic poles S_1 and S_2 is

$$S^2+(A_1/A_2)\,S+1/A_2 = 0. \tag{29}$$

The effect of S_3 will only be important at frequencies above the roll-off; the natural frequency $f_n = \omega_n/2\pi$ and damping factor ζ are then given by

$$\omega_n^2 = 1/A_2, \quad \zeta = A_1/2\surd A_2, \tag{30}$$

i.e.
$$\left.\begin{aligned} \omega_n^2 &= \frac{2R_cR_W(\phi-1)}{(M'C\phi\beta/KH)R_1R_2+(R_cL_W-R_aL_b)},\\ \zeta &= \frac{C\beta\phi R_1R_2}{2HK(2R_cR_g\phi(\phi-1))^{\frac{1}{2}}\,[(M'C\beta\phi/KH)\,R_1R_2+(R_cL_W-R_aL_b)]^{\frac{1}{2}}},\end{aligned}\right\} \tag{31}$$

where $R_1 = R_a+R_{WL}$, $R_2 = R_{bt}+R_c$ and the constant-current time constant T has been replaced by $C\beta\phi/H$ and α by $2R_g\phi(\phi-1)$ to separate the roles of the overheat ϕ, wire thermal capacity $C\beta$ and wire heat-transfer coefficient H.

For moderate values of the a.c. gain K, both terms in A_2 [the denominators of (31)] are significant. In this case, the important controllable anemometer parameters include M', K and L_b. It can be seen that ω_n is increased by increasing the amplifier bandwidth (reducing M') or its gain. However, whilst increasing K will reduce the damping, reducing M' will increase it.† The natural frequency is increased by increasing L_b, but the damping also increases up to a point beyond which the system becomes unstable.

The operating conditions H and ϕ and the probe characteristics R_g and $C\beta$ also have a strong influence. Increasing the wire heat-transfer coefficient H (by

† Some effects of the amplifier bandwidth on system stability were discussed by Grant & Kronauer (1962).

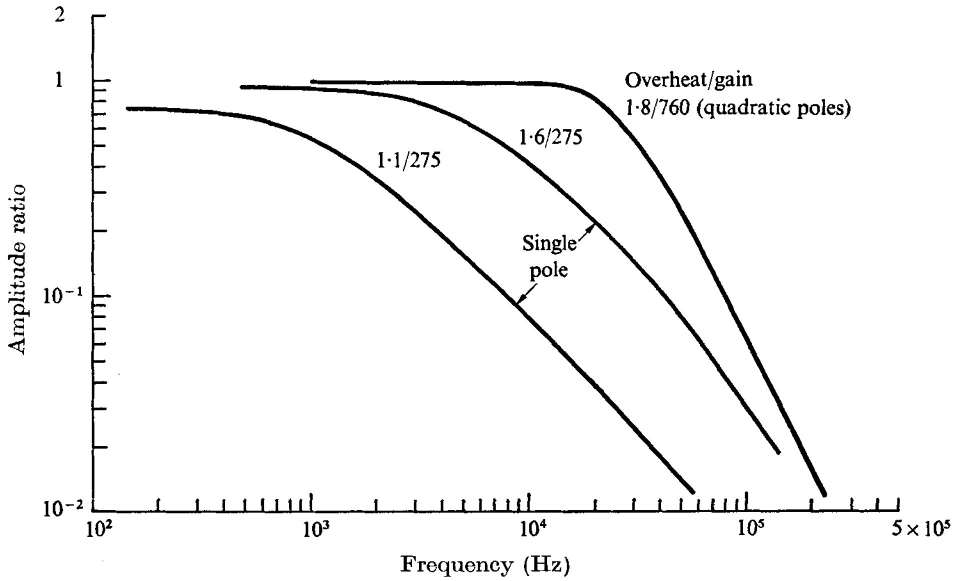

FIGURE 2. Predicted frequency response.

increasing the fluid flow past it) will increase the natural frequency, and also the damping. A similar effect is caused by increasing the overheat ϕ (which, to a first approximation, will not affect H).

If it is desired to operate with uniform frequency characteristics under different operating conditions, the anemometer parameters may be altered to compensate. For example, the gain may be varied to compensate for changes in H directly, since it always occurs in the combination HK. Compensation for changes in overheat may also be achieved by varying the gain, but not as accurately as in the case of flow-condition changes.

For values of the a.c. gain of order 10^3, further increases in K have no effect on ω_n, but will continue to decrease the damping. Also, the amplifier bandwidth becomes unimportant. In this case, the effect of the inductance L_b of the compensating inductor is even more important than at low to moderate gain. Compensation for changes in flow conditions, which now affect only the damping, may still be achieved by altering K, but control of L_b is more useful to adjust for changes in overheat.

Figure 2 shows the fluctuation response at near optimum setting for an overheat of 1·8, obtained with a gain of 760.

4.2. *Operation with high damping*

A different facet of the system behaviour is seen when the damping is greater than optimum and the frequency response is restricted, these effects being the result of reducing both the gain and overheat. This mode of operation has been found necessary in the wet steam turbine measurements which prompted the present investigation (Wood 1973). It has been found desirable to limit the overheat in order to prevent overheating in parts of the wire when water droplets alight on it, and the damping was increased to improve stability during periods when the wire was bombarded with intermittent showers of droplets. In this mode, one of the poles of the anemometer equation is smaller than the others, so that the system roll-off is determined by a single pole, given by (28). However, the next two larger poles are usually within an order of magnitude of the smallest, so that the response to frequencies a little above the roll-off is influenced by all three poles. The response at frequencies beyond the roll-off is of interest in the present case, because of the need to correct or compensate for it.

The principal part of the response is thus characterized by a single curve, with the frequency at the -3 db point given by ω_n. Substituting for T and α in (28), we have

$$\omega_n = \frac{1}{A_1} = \frac{2KH(\phi-1)\,R_g R_c}{C\beta R_1 R_2}. \tag{32}$$

Clearly the amplifier gain is the dominant anemometer parameter in this case and it is of direct use in compensating for changes in flow conditions or overheat.

The responses with high damping (gain = 275) at overheats of 1·6 and 1·1 are shown in figure 2. These curves also indicate how the response is cut if the overheat is reduced at fixed gain.

5. Comparison of theoretical and experimental results

The calculations were carried out with values of the anemometer parameters appropriate to the DISA 55D01 system, which were as follows:

$$R_a = 50\,\Omega, \quad R_c = 1\,\mathrm{k}\Omega, \quad R_{bt} = 20R_{WL}, \quad R_g = 3{\cdot}5\,\Omega, \quad R_s = 28\,\mathrm{k}\Omega.$$

Typical values of the bridge inductances are $L_W = 2\,\mu$H and $L_b = 30\,\mu$H. The probe lead resistance was 0·7 Ω and a typical 5 m cable resistance would be 0·5 Ω, giving a total R_L of 1·2 Ω; in many of the tests, the probe constructed by the author for turbine measurements, with a cable resistance of 1·5 Ω and $R_L = 2{\cdot}2\,\Omega$, was used.

The d.c. gain was 45000, the a.c. gain K could be varied in fixed, calibrated steps from 145 to 4700, and the amplifier bandwidth could be varied in three fixed steps to give -3 db points at approximately 120, 250 and 600 kHz (the middle setting was used throughout the present investigation). The inductance L_b of the compensating inductor was continuously variable over a moderate range but was uncalibrated.

The calculations quoted have all been made for a standard probe heat-transfer coefficient, giving $E_0 = 3{\cdot}2$ V at an overheat of 1·6. This would be given by steady air flow at 5 ms^{-1} at atmospheric conditions, and is typical of the values of H obtained in the low density steam flows which have been mentioned.

5.1. *Sine-wave response*

Figure 3 shows a comparison between the experimental and theoretical sine-wave responses for high and low overheats at moderate and high gain respectively. The agreement is excellent at moderate gain, but worsens at high frequency for the

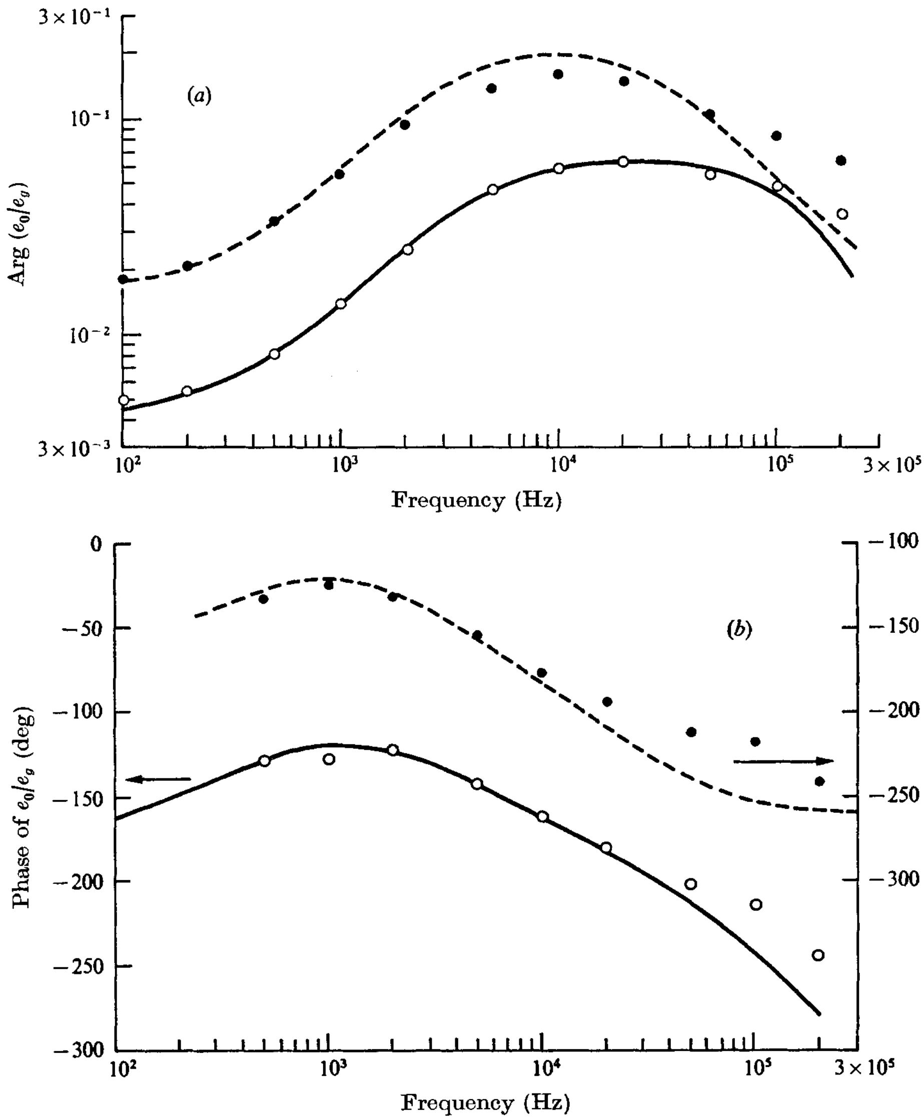

FIGURE 3. (a) Comparison of experimental and theoretical response of anemometer to sinusoidal test signal. Equivalent air velocity = 5 ms^{-1}. ●, overheat = 1·1, gain = 1100; ○, overheat = 1·6, gain = 275. (b) Corresponding phase responses.

higher gain, at this low overheat, mainly because of the increased importance of the term $R_c L_W - R_a L_b$. This is because the probe and compensation inductances vary with frequency owing to the 'skin effect' (Davies, private communication); the variations of L_W and L_b are dissimilar, so that their relative magnitude alters. Also any stray bridge reactance will affect this term, and increase the error.†

† It appears that at low input levels the gain of the amplifier in the DISA anemometer varies with changes in voltage level; however, there was no evidence that this was a factor in the present experiments, although such behaviour would help to explain the discrepancies at low overheat.

The most satisfactory agreement is obtained by selecting values of the inductances appropriate to a moderately high frequency, in this case $L_W = 1{\cdot}8\,\mu$H and $L_b = 30{\cdot}0\,\mu$H.

The corresponding fluctuation responses are shown in figure 5 (curves 1 and 6). An improved calculation of the fluctuation response could be obtained by dividing the measured values of the sine-wave response by the transfer function $[(TD+1)(R_{WL}+L_W D)+\alpha]/R_s$ to obtain $K_2(E_1 D+1)/f_L$. [See (7) and (25).]

5.2. *Square-wave response*

Whilst use of the square-wave test is invaluable as an aid to setting up the anemometer, the rules frequently given to obtain the frequency response from it are liable to be in error. The square-wave response is strongly affected by the higher-order terms in the anemometer equation, and some of the rules are derived from approximate theories, which omit these terms. In the following paragraphs the various methods are described and their limitations are pointed out.

One popular method is to measure t_e, the width of the response pulse at the point where it has decayed to $1/e$ of the peak value. The frequency at which the fluctuation response is attenuated by 3 db (the 3 db point) is assumed to be $1/2\pi t_e$, and the response is thereafter said to fall at 6 db per octave. However, this is only true if the governing equation for the system is of first order, and it has been shown here that such an approximation is valid only in certain restricted circumstances.

Another method was given by Freymuth (1967). He suggested that the frequency f_{-3} at the -3 db point may be obtained from the square-wave response through the relation

$$f_{-3} = 1/1{\cdot}5t_3, \tag{33}$$

where t_3 is the time from the beginning of the test pulse until it has fallen to 3 % of its maximum value. Equation (33) may be applied to the experimental square-wave response in figure 4, which was judged to be near optimum. The overheat was 1·8 with a gain of 760, corresponding to the fluctuation response shown in figure 2.

If t_3 is measured from the experimental square-wave response of figure 4, inserting the value into (33) gives $f_{-3} = 23$ kHz. By contrast, the corresponding value obtained from the theoretical fluctuation response of figure 2 is 31 kHz. It has been shown in § 5.1 that the theory presented here is adequate to calculate the fluctuation response, at least up to 100 kHz, so that the latter frequency is considered to be accurate.

As a matter of interest, the square-wave response predicted by the present theory for the case cited is also shown in figure 4. Here again, of course, the agreement is affected by the frequency dependence of L_W and L_b, by bridge capacitances and by stray reactances, i.e. by the inaccuracy of higher-order terms. The high frequency theoretical response, corresponding to small times in the square-wave response, is thus in error. The main effect here is a time shift; the shapes of the curves when overlaid are in very close agreement. Of the causes of the inaccuracies mentioned above, the frequency dependence of the inductance is

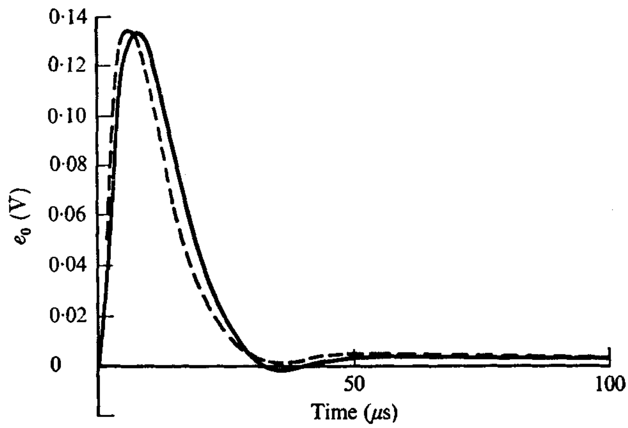

FIGURE 4. Square-wave response at optimum setting in air at 5 ms^{-1}. Overheat = 1·8, gain = 760, E_g = 1·0 V. ——, theory; ------, experiment.

dominant. The time shift shown in the figure is eliminated, but the height of the peak is increased, if the response is calculated with inductances appropriate to higher frequencies. However, the associated frequency response at moderate frequencies, including the roll-off, is then in error. This example illustrates the point that even a sophisticated anemometer theory may not accurately predict the square-wave response. Nevertheless, if the present theory is used with inductance values appropriate to moderately high frequencies (say 50 kHz), the frequency response may be obtained more rapidly and with more certainty than with any experimental technique.

A further possibility for interpreting the square-wave response is to assume that the system behaviour is described to a good approximation by second-order response characteristics (§ 4.1). The square-wave response which is judged to be optimum is then taken to have a damping factor of 0·7. The shape of the fluctuation response is known and is related to some feature of the square-wave response. Similar considerations led to Freymuth's relation.

However, the method is inhibited by two factors. First, it is difficult to judge the damping factor accurately from the square-wave response and second, as already mentioned, the square-wave response itself will be particularly affected by the higher-order terms in the anemometer equation at small times, including the time at which the peak occurs. Thus, whilst the fluctuation response may be well approximated by a second-order characteristic, the real square-wave response may not, so that its interpretation on this basis is unreliable.

An alternative method purports to obtain ω_n by assuming that it is the frequency obtained when the square-wave response is adjusted, by reducing the damping, to make it 'ring'. However, when the response is restored to optimum damping, the natural frequency will also change significantly in any practical system.

It is, therefore, unwise to use the square-wave test as anything but a qualitative aid to tuning the anemometer.

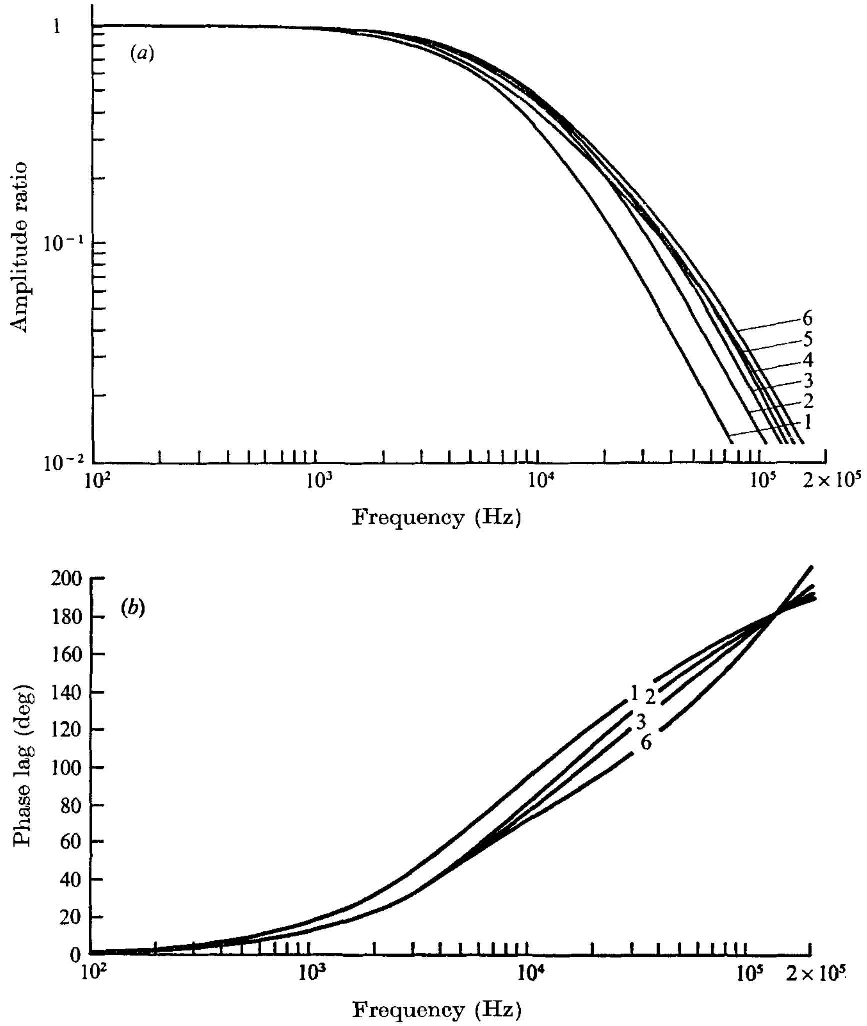

FIGURE 5. (*a*) Compensation for variation of frequency response with overheat by variation of gain. Amplitude response (for key see table 1). (*b*) Corresponding phase responses.

6. Equalization of frequency response at different overheats

It was suggested in § 4 that the variation of frequency response with overheat could be compensated for by simultaneous variation of the gain, particularly in the high damping case which was of interest in the present investigation. Figure 5 shows the results obtained with the fixed gain settings of the DISA 55D01 anemometer at overheats between 1·1 and 1·6 for an atmospheric air flow of 5 ms^{-1}. The degree of improvement compared with results for fixed gain may be seen from a comparison with figure 2. The characteristics of the curves are listed in table 1. For overheats between 1·2 and 1·6 and frequencies below 30 kHz the agreement of the amplitude responses is extremely good, and a single compensation characteristic would be feasible for many experiments. The improvement in

Curve	Overheat	Gain	−3 db point (kHz)
1	1·1	1100	4·3
2	1·2	760	5·7
3	1·3	540	5·8
4	1·4	390	5·2
5	1·5	275	4·6
6	1·6	275	5·6

TABLE 1. Characteristics of curves in figure 5

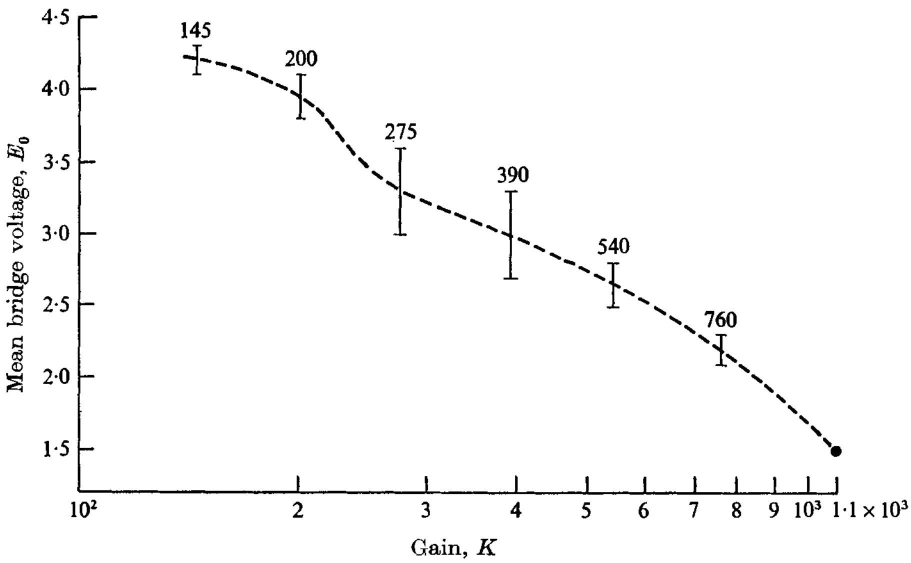

FIGURE 6. Gain required to equalize frequency response with variation of steady bridge voltage.

phase response is less good above 10 kHz, but the use of Kovasznay's method (see § 1) allows this to be ignored.

Further improvement could be obtained by variation of L_b and clearly it would be useful if the control for this parameter were calibrated.

As indicated in § 4, the frequency response is also affected by changes in flow conditions, which may be directly compensated for by changes in gain. In fact both the flow conditions and the overheat are sensed by the anemometer system through the current drawn by the wire and hence through the bridge voltage E_0. Thus the results shown in figure 5 may be extended so that equalization of the frequency response may be achieved with variation of E_0, this variation being due to changes in overheat or in flow conditions. To obtain the response of the standard form chosen for figure 5, the required gain variation with variation of E_0 is shown in figure 6. The values of the gain shown in the figure are the fixed steps available on the anemometer used in the tests; with continuous variation of gain, a continuous curve could be obtained.

Thus, the scheme adopted is to set the anemometer operating in the flow, to measure E_0, and then to set the gain required to give the selected standard response by reference to figure 6.

7. Conclusions

The anemometer theory of Perry & Morrison (1971*a*) has been extended to include the effect of amplifier bandwidth and of probe lead and trim resistances. The extended theory has been applied to an anemometer system which employs high d.c. gain to force the bridge into balance, coupled with variable a.c. gain to control the frequency response. A direct test has been devised for the theory, the results of which suggested that the theory was sufficiently accurate to enable the response to be calculated. Limitations were imposed, however, by the variation of bridge inductances with frequency, and by stray reactances.

If it is required to vary the overheat for measurements in flows containing both temperature and mass-flow fluctuations, the resulting variation in frequency response may be largely eliminated by simultaneously varying the gain of the servo amplifier. The same means may be employed to maintain a constant frequency response with variation of mean flow conditions.

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